Evidence Bulletin

National Centre for Excellence in the Teaching of Mathematics

How can you use research evidence to enhance your mathematics teaching?

It is 20 years since the publication of 'Better Mathematics' (HMSO, 1987) and 25 years since the publication of 'Mathematics Counts' (HMSO, 1982). Each of these documents articulated views on what constituted effective learning of mathematics – informed by accumulated research findings and interpreted through the prevailing culture and values.

Working with leading researchers, trainers and teachers, the National Centre for Excellence in the Teaching of Mathematics is undertaking a year-long review of current attitudes, practices and influences across today's mathematics education.

In this evidence bulletin, we take as our starting point some of the key principles for effective teaching of mathematics[1]:

- build on the knowledge learners bring to sessions
- expose and discuss misconceptions
- develop effective questioning
- use cooperative small group work
- emphasise methods rather than answers
- use rich, collaborative tasks
- create connections between mathematical topics
- use technology in appropriate ways.

The bulletin explores each of these principles by presenting summaries of research evidence, sometimes illustrated with case studies or combined with practical activities to help you translate the findings into classroom practice.

All our articles use research as a starting point, and generally the most recent evidence. We have used evidence from all phases and levels: primary, secondary and post-16, and we point out the relevance and implications of the research for all teachers of mathematics.

We provide full reference details including weblinks where available, so that you can look up and read the original research for yourself. You'll also find very useable supplementary resources, including more detailed summaries of the research plus classroom activities.

We hope you like it …

[1]Improving Learning in Mathematics, DfES Standards Unit (2005)

In this Issue

Improving Learning in Mathematics, DfES Standards Unit (2005)

Student motivation

How can we encourage students to work hard at mathematics

How to motivate students to work hard at mathematics, particularly those who struggle with the subject and are in lower sets, is a problem that's high on many teachers' agendas. Whilst some students strive to improve at mathematics, we're all well aware of others who see little point in putting effort into learning the subject. What strategies are effective at motivating these students?

A systematic review of research[1] undertaken by researchers at York University which looked into the issue found a range of strategies with potential for encouraging students to work harder. These included strategies that enabled students to see themselves as 'mathematicians' – that is people who can understand and do mathematics, and innovative strategies, such as using ICT. On this page we introduce you to some of the lively and positive ways of teaching the reviewers found – ways that will support, engage and challenge all your students, and increase their understanding too.

How can teachers help students see themselves as mathematicians?

The reviewers found that strategies that helped students to feel positive about their mathematical ability, included:

- providing a caring and supportive classroom climate
- enabling all students to feel equally valued
- providing activities which students find challenging and enjoyable
- enabling students to gain a deeper understanding of mathematics
- providing opportunities for students to collaborate.

On several pages of this *bulletin* you'll find we point out exciting web resources that will give you fresh ideas for making mathematics challenging, relevant and enjoyable, whilst on pages 6-7 we show you how you can help your students develop their mathematical understanding through working effectively together in small groups.

What else helps?

The reviewers found that using ICT (such as, interactive whiteboards, software packages and calculators) can have a powerful effect on how hard students work. The reviewers noted two kinds of motivating effects:

- using ICT to make lessons enjoyable
- using ICT in a way that enhanced deeper understanding of mathematics.

The reviewers pointed out that whilst both aspects were important, enjoyment isn't enough on its own. If you turn to page 8 you'll find out how you can use whiteboards in ways that enhance students' understanding as well as make learning mathematics more fun.

Other methods that help, according to the review, include:

- the use of mental/oral starter sessions and whole-class interactive teaching (as primary schools do during the 'numeracy hour')
- cognitive acceleration in mathematics education (CAME) or CAME-type lessons
- assessment for learning (AfL) practices, such as asking questions that probe students' understandings and giving students time to think deeply about their answers.

We take a look at the powerful effects of CAME lessons on the next page, whilst on page 9 we explore ways you could enhance your questioning skills.

Why is professional development so important?

It's hard making changes to your practice on your own. The reviewers stressed that teachers need ongoing support and training when they try out the strategies they outlined. This is because in order to implement them effectively, you need to have a good understanding of the underpinning rationale and a good command of the skills and techniques they need. They suggested that a good form of professional development was working collaboratively with colleagues. Working together can help you to explore and evaluate changes to your practice that have a positive effect on students' motivation to work hard.

How could you put this research to work?

The review found that teaching styles and methods which helped students see themselves as people who can understand and do mathematics, really could make them work harder. Motivating strategies included providing enjoyable, challenging activities, giving students opportunities to collaborate, and using ICT and other innovative methods such as CAME and AfL practices.

- Could you investigate ways of encouraging all students to feel they can do mathematics, by, for example, helping them to work together and support each other on problem-solving activities?
- Could you make more use of ICT to deepen students' understanding of mathematics, for example, by using dynamic graphical software to illustrate the effect of changing coefficients on the graphs of functions?
- Could you work with your colleagues to evaluate the impact of innovative strategies such as CAME on your students' motivation? Perhaps you could consider which aspects of the CAME lessons are effective in your opinion and why this is?

References and resources

[1] Kyriacou, C., & Goulding, M. (2006) A systematic review of strategies to raise pupils' motivational effort in Key Stage 4 Mathematics. In: Research Evidence in Education Library. London: EPPI-Centre, Social Science Research Unit, Institute of Education, University of London. Available at: http://eppi.ioe.ac.uk/cms/Default.aspx?tabid=714

A more detailed summary of this research is available on The Research Informed Practice Site (TRIPS) at: www.standards.dfes.gov.uk/research/themes/numeracy/

Cognitive acceleration

How can we enhance students' thinking in mathematics?

In a culture of high-stakes testing and accountability, it is heartening to know that teaching thinking skills in mathematics can improve examination results in the long term. Rather than focusing on students' knowledge of mathematics procedures, the Cognitive Acceleration in Mathematics Education (CAME) project aimed to enhance students' general thinking ability. CAME integrated thinking with collaborative learning. This took place primarily in thirty specially designed 'thinking maths' lessons. Project teachers used these lessons about once a fortnight over a two-year period, but were encouraged to use the same teaching skills in other mathematics lessons too.

The effects of the CAME approach can be dramatic. Researchers found that students who had been taught using the CAME approach in the first two years of secondary school, later gained, on average, 0.8 of a GCSE grade compared with other students whose teachers had not used the approach. Higher ability students, in particular, made substantial gains – in some classes, the proportion of students gaining a C-grade or above doubled. The CAME approach has been used successfully at primary level too. For example, around 40% of pupils from classes that had been well below the national average before the programme of lessons in Year 1 were at the 2B level by the end of Year 2 compared with only 20% of pupils from classes at the national average.

References

Shayer, M. & Adhami, M. (2007) 'Fostering cognitive development through the context of mathematics: results of the CAME project'. Educational Studies in Mathematics 64 pp. 256-291

Adhami, M., Johnson, D.C. & Shayer, M., (1999) 'Tensions in planned mathematics lessons' in Bills, L (ed.) (1999) Proceedings of the British Society for Research into Learning Mathematics 19 (3), pp. 35-41. Available at: http://www.bsrlm.org.uk/IPs/ip19-3/index.html

What do CAME lessons look like?

CAME 'thinking maths' lessons involve a cycle of three main activities:

- Concrete preparation, in which the class begins to establish a shared understanding of the lesson theme, by discussing it in pairs and as a whole class.
- Collaborative learning, when the pupils work for 10-15 minutes in pairs or small groups on a task with the expectation of having to explain their ideas to the rest of the class. During this second phase, the teacher does not 'help' the groups, (although s/he might ask a strategic question if a group seemed stuck), but listens and notes where each group has reached, to help plan the order in which each group will contribute to the whole-class discussion which follows.
- Whole-class discussion, which starts when the teacher judges that a variety of ideas have emerged in at least some of the groups. It isn't necessary for all the students to have completed their solutions. Each group reports its ideas (or focuses on specific ideas at the teacher's request) and the teacher encourages other students to ask questions. The teacher's role is to manage the students' interactions with each other. During this phase, each student has the chance to extend his/her understanding, even if the student's own group has not reached a solution.

Is it important how you group students for CAME lessons?

The researchers noticed how students in 'remedial' sets gained little from CAME lessons despite being taught by good CAME teachers who achieved excellent value-added results with other classes. They suggested the reason for this was that such groups lacked intermediate ability students who could provide higher-level insights in the discussions.

An example of a discussion from a Year 5 CAME lesson

CAME lessons focus on exploring students' reasoning though discussion. Teachers ask open questions and use pupils' responses to help build a shared understanding. The open questions the teacher asks can provoke lots of different responses, including misconceptions and conceptions that are valid, but are presented in informal or unusual ways. Teachers need to be good listeners and to be aware of a range of common misconceptions if they are to use pupils' responses to help build a shared understanding.

Teacher: You each have an isosceles triangle. What else do you notice about it? See what patterns you can make with it.

Pupil 1: (With agitation): This is not isosceles. It is scalene. (She turned the shape in her hand, touching the sides, and repeated her indignant phrase.)

Teacher: Why do you think it is scalene, not isosceles?

Pupil 1: (Holding the shape with the long side horizontally in one hand and running fingers from either end of the base to meet higher up in the air.) Isosceles will be there.

Pupil 2: (Running a finger over the two equal sides of the plastic triangle.) These two sides are the same.

Pupil 1: (Suddenly changes her mind.) Ah, yes it is an isosceles.

Teacher: Can you tell me why you changed your mind? That is a useful thing to know.

Pupil 1: I have always seen isosceles with equal longer sides upwards.

You can find out more about managing group discussions effectively on pages 6-7.

Calculation methods

Why is it important to build on pupils' intuitive ways of working?

You've spent a long time showing your pupils how to do a particular calculation, and given them plenty of examples to practise, yet many of them get the answers wrong when they attempt similar questions in a test. What would help? A recent study1 of how pupils approach division showed that it could be helpful to teach a structured written method of recording that builds upon pupils' intuitive understanding and experience gained from using their own informal methods.

The researcher reached this conclusion after exploring the strategies Year 5 pupils were taught for tackling division problems and how successful the pupils were at using the strategies. She found that the strategies the pupils used varied widely from school to school:

- in some schools, pupils mostly used partitioning strategies in which the digits were split into tens and units for the calculation
- pupils in some schools mostly relied on informal, unstructured methods to calculate and record their answers and rarely used any type of structured or formal written methods of recording
- in some schools, pupils used the 'chunking' method, but in other schools they didn't use the method at all. (Chunking is a method for decomposing numbers using known relationships. The sum 432 ÷ 15 can be chunked as 300, 60, 60 and 12 with chunks subtracted from 432, so that each time the total to be divided is reduced to make the division by 15 easier. A less efficient way for decomposing t he numbers would be to repeatedly subtract 15).

Which calculation methods were the most successful?

The relationship between pupils' use of, and success with, different strategies for division was complex. The researcher found that different approaches seemed to suit different pupils and some differences between boys and girls also emerged. What was important was that pupils were able to use approaches that suited the numbers in the problem, and could make a structured, formal written record of their calculation.

-The high scoring schools (according to the researcher's test) used different methods, including the traditional algorithm (a method of recording based on partitioning the whole number into hundreds, tens and units etc, sometimes referred to as the 'bus-shelter method'). The Year 5 pupils in schools that continued to use mainly informal methods to approach division problems and which delayed teaching any kind of formal way of recording, were less likely to do well in tests.

What differences emerged between boys and girls?

The researcher noticed that for some problems, boys preferred to calculate mentally and were less likely to use written methods than girls. Girls tended to use inefficient strategies and they particularly benefited from being supported in developing a structured method of recording. Boys' overall performance was better than the girls' (although not in all schools). In schools where girls did better, most of the girls used the chunking method at least once.

Does practice make perfect?

A study[2] which explored how young pupils' understanding of addition and subtraction was related to their competence at calculating, found that conceptual understanding and skill in computation were independent of one another. The researchers noticed how some pupils' understanding of what was involved in a problem outstripped their numerical ability to calculate it correctly. This led the researchers to conclude that pupils' understanding of mathematical concepts did not develop from familiarity with practising sums.

How could you put this research to work?

The first study found that pupils need help to move from informal methods of calculation to making structured written records. It was important that pupils could use different approaches. The second study showed that simply practising sums did not help pupils to understand the concepts underpinning them.

- Would asking your pupils to explain the method they used to reach their answer to other pupils, and discussing the approaches with the whole class help to focus their attention on efficient ways of recording?
- Could you share with your pupils a wider variety of calculation strategies?
- Might discussions with small groups of pupils about why they choose to use a particular method throw any light on how you can best help them?

References and resources

[1] Anghileri, J. (2006) A study of the impact of reform on students' written calculation methods after five years' implementation of the National Numeracy Strategy in England. Oxford Review of Education 32 (3) pp.363-380

[2] Gilmore, C., & Bryant, P. (2006) Individual differences in children's understanding of inversion and arithmetical skill. British Journal of Educational Psychology 76 pp. 309-331

More detailed summaries of these studies are available on The Research Informed Practice Site (TRIPS) at: www.standards.dfes.gov.uk/research/themes/numeracy/

Effective mathematics teaching

What makes some teachers of mathematics more effective than others?

There's growing evidence that we are all much better at teaching mathematics than we used to be. But what is it that's making the difference? Researchers[1] have explored the characteristics of teachers whose classes made greater gains in mathematics. Although their study was published ten years ago, their findings are just as relevant and important today. The researchers found that amongst other things, effective teachers:

- connected different areas of mathematics and different ideas in the same area of mathematics
- believed it was important that pupils knew different calculation methods and were able to choose the most efficient method for the problem in hand
- probed pupils' reasoning to help establish and emphasise connections and address any misconceptions
- encouraged purposeful discussion in whole classes, small groups and with individual pupils.

These are themes that crop up over again in the research evidence. For example, we looked at the value of discussion in the article about cognitive acceleration on page 3 whilst on page 4 we showed the value of pupils knowing different ways of calculating. On this page we show you two ways you could gather evidence about your own teaching based on the findings above, and how you could use the evidence to further enhance your own practice.

How do you help your pupils connect ideas in mathematics?

The study found that effective teachers believed it was important to encourage pupils to make connections between different ideas in mathematics. One way of doing this is to encourage pupils to explore a concept in different kinds of real life situations.

You might like to explore how you help your pupils recognise similar questions when they appear in different guises. You might find it helpful to arrange for a colleague to observe a lesson where, for example, you planned to ask a set of similar questions in different forms or to review a video recording with you. Some of these could be oral and some written. You could discuss with your colleague how well your pupils identified questions that required similar solution strategies.

Could you do more to explain to your pupils how questions requiring similar solution strategies can look different, for example through:

- a change of cover story – questions using the same format and similar numbers etc, but a different narrative
- a different look – such as questions in multiple choice form
- different words – for example, using the word dozen instead of 12
- a different question structure – for example instead of asking how much a person has spent, asking how much change was given
- a wider scope – such as including a greater number of items to include in the calculation.

How can working with their peers help pupils to learn?

The study found that where teachers believed that the best approach to help pupils learn mathematics was individualised practice and problem-solving, they were less effective. Because the pupils worked individually, they did not become aware of the approaches other pupils used, which narrowed the pupils' sense of possibilities. The more effective teachers were convinced of the value of pupil discussion. Working with their peers allows pupils to see different ways of tackling the same task and, as a result, extend their own repertoire. Pupils also come to learn new and better strategies through trying to explain the strengths and weaknesses in their own work to others.

You might like to explore the learning that takes place between pairs or small groups of pupils in one of your classes. You could monitor an activity specifically designed to help your pupils learn from each other (perhaps one of the activities suggested on the back page). You could observe a group, listening to and noting down the strategies they suggest. You could also ask your pupils to share the various strategies they explored in their groups with you and the rest of the class during a plenary session.

Having gained an idea of strategies your pupils use, you might like to consider how you might help to enhance them. For example, you could ask your pupils to reflect on the strengths and weaknesses of each other's strategies, noting perhaps quick and efficient ways of working. Could you share ideas with a colleague about ways of enabling pupils to work effectively with each other?

References and resources

[1] Askew, M., Brown, M., Rhodes, V., Wiliam, D. & Johnson, D. (1997) Effective teachers of numeracy. Final report. London: King's College London.

You can read a detailed summary of the report on the GTC Research of the Month (RoM) website at: www.gtce.org.uk/research/rom-topics/rom_curriculum/numeracy1/

Structured group work

How can we help pupils discuss mathematical problems together effectively?

How often have you asked your pupils to work in pairs or in small groups to discuss a mathematics problem or investigation, only to find that they argue or spend the time chatting about other things? There is excellent research evidence about the effectiveness of structured group work for pupil learning, so what can you do to promote such focused, productive discussions? We've found two recent studies that explored this issue.

One study[1] focused on a teaching programme called 'Thinking Together' that aimed to increase the amount of 'exploratory talk' pupils used through a series of 12 lessons. Exploratory talk is an educationally effective kind of talk that involves pupils explaining and justifying their ideas before reaching an agreement. The researchers found pupils in the target classes significantly increased the amount of exploratory talk they used in their group discussions and improved their attainment in mathematics significantly more than those in the control classes. Crucial to the success of the approach was the way their teachers guided and modelled the pupils' use of language.

How did effective teachers guide and model group discussion?

The teachers spent the first few lessons raising the pupils' awareness of exploratory talk and negotiating a set of ground rules for group discussion, such as:

- ask what people think and what their reasons are
- include everyone
- listen to each other
- reach an agreement before deciding.

In later lessons, the pupils were expected to apply their developing discussion skills to the study of mathematics. During this time, the effective teachers modelled the ground rules for exploratory talk in their whole-class dialogues. As they led the pupils through an activity, they invited as many pupils to speak as possible, asked them to give their reasons, respected their contributions and finally ensured agreement was reached.

What activities work best for group work?

Choosing the right kind of activities for collaborative discussion is as important as the kind of dialogue you use. In another study, the researcher gave 48 college teachers of GCSE mathematics re-sit classes a set of resources for teaching algebra which included discussion-rich classroom materials for ten lessons. One set of activities, for example, encouraged students to reflect on common generalisations. Collaborative discussion was encouraged through asking each group to make a poster. This involved pasting down statements (expressed in words or symbols) under the headings 'always true, sometimes true or never true' and surrounding the statement with justification and explanation.

For example:

$12a>12$

If you multiply 12 by a number, the answer will be greater than 12

Another kind of activity involved asking the students to create their own problems which other students were invited to solve. The originators and the solvers then worked together to see where difficulties had emerged.

As with the first study, the teachers who encouraged their students to discuss and re-formulate their own ideas were much more successful at improving their students' learning and maintaining their confidence and motivation than those teachers who simply conveyed facts and skills.

Example of how a teacher modelled the ground rules for group discussion

In this lesson, the teacher was preparing the class for a computer-based activity in which they were asked to consider what operation could be carried out on a number in order to end up with another (eg. divide it by 2 and add 2).

Teacher: OK. I'm going to put a number in *(looks at class quizzically).*

Louis: One thousand.

Teacher: OK, Louis had one thousand. Anybody think yes or no to that idea? David.

David: Start off with an easier number.

Teacher: Start off with an easier number. By an easier number what kind of number do you mean?

David: Um. Something like, lower, five.

Teacher: Fine. A smaller number, a lower number, yes. Louis can you see that point of view?

Louis: Yes.

Teacher: If we put in a thousand we could end up with a huge number. If we put in five do you think it will be easier to work out what the machine has done?

Class: Yes.

Teacher: Everyone agree?

Class: Yes.

Teacher: OK, I'm going to type in five.

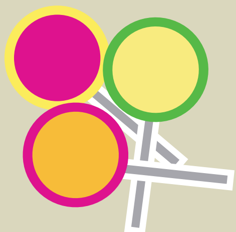
You may also like to read the example of how a teacher explored pupils' reasoning in the article about cognitive acceleration on page 3.

Where might you find discussion rich activities you could use for group working?

There are many websites that make these kinds of resources freely available. The activities pictured here came from Cambridge University's NRICH website *(www.nrich.maths.org/)* which has a host of investigations suitable from early years right through to A level. The dialogues your pupils have with each other as they work on the tasks should be well worth listening in on.

Take Three from Five

Can you guarantee that it is always possible to choose three numbers that will add up to a multiple of 3 from ant set of five positive whole numbers? Can you explain why?



Lots of Lollies

Frances and Rishi were given a bag of lollies. They shared them out evenly and had one left over.

Just as they had finished sharing them, their friends Kishan, Hayley and Paul came along. The newcomers wanted some lollies too, so the children shared them out again. This time they had two lollies left over. How many lollies could there have been in the bag?

Could you help your pupils to work more effectively together?

The studies showed the value of structured group work and the importance of pupils sharing their reasoning with each other. You may like to review how your pupils conduct group work using the questions below as a framework to analyse what happens. Or you might like to ask a trusted colleague to observe a group work activity in one of your lessons and give you feedback about the following items:

- Do pupils take turns or do they frequently talk over each other or interrupt?
- Do they invite contributions from each other?
- Do they listen to each other, and respond and react to each other's contributions?
- Do they invite each other to offer explanations (eg. by asking 'Why do you think that?')
- Do pupils elaborate their contributions by providing reasons, explanations, and examples?
- Do pupils modify what they say in the light of each other's comments?
- Do they pool ideas before reaching a group decision?

You could use yours or your colleague's observations to help you identify your pupils' strengths and weaknesses in group working. Or you could use your evidence as a focus for a discussion about group work with your pupils. Would you find it helpful to work with your pupils to build rules for structuring group discussion and observe the effects of these rules on a subsequent group work activity?

How might you enhance the way you guide and model effective talk with your pupils?

The studies showed how effective teachers guided and modelled exploratory talk. They asked pupils to give reasons for their views to explore their thinking and understanding, rather than check their pupils' recall of factual knowledge.

Gathering evidence about the way you interact with your pupils could help you to discover the extent to which you create opportunities for pupils to share their thinking and understanding. One way you could do this would be to record a teaching session on audiotape. When you listen to the tape, you could note the different kinds of questions you ask:

- *factual questions*, that test knowledge and recall (eg. "What are the properties of an isosceles triangle?")
- *speculative questions*, that invite pupils to offer their ideas, opinions or hypotheses (eg. "If you did x, what do you think might happen?")
- *process questions* that invite pupils to explain their thinking or make what they understand explicit (eg. "How did you work that out?" Or, "Can you explain why?)
- *procedural questions*, that help you to manage the discussion (eg. "What do you think Clare?" Or, "Does anyone disagree?")

You could then sort the questions you asked into the different types listed above and look for any patterns. If for instance, you find you ask few speculative questions, you might like to work with a colleague to come up with different ways of asking this kind of question that you could use in another lesson. You could also monitor how your pupils respond.

References and resources

[1] Mercer, N. & Sams, C. (2006) Teaching children how to use language to solve maths problems. *Language and Education 20 (6) pp. 507-528*

Further information about the Thinking Together approach, including teacher ideas and materials is available at: *www.dialoguebox.org and http://talking-for-success.open.ac.uk*

[2] Swan, M. (2006) *Collaborative learning in mathematics: A challenge to our beliefs and practices'* NRDC and NIACE. (A CD of resources and video clips is available with the book).

Interactive whiteboards

How can whiteboards help to make learning interactive?

Most of us use interactive whiteboards (IWBs) in our teaching these days. One reason they've become so popular is because they enliven learning through interactive presentation techniques such as:

- drag and drop – where an on-screen item can be moved for the purposes of comparison and testing hypotheses etc
- colour, shading and highlighting to emphasise similarities and differences
- hide and reveal – hiding then opening a response once an idea has been understood, to step ideas and the development of hypotheses
- animation to demonstrate principles and illustrate explanations.

But does the technology support interactivity in terms of enhancing our interactions with and between pupils to develop understanding? Interactive teaching in this sense, involves teachers and pupils:

- working together on a learning task
- listening to each other to share ideas and consider alternative viewpoints
- building on their own and each others' ideas and chaining them into coherent lines of thinking.

To find out whether IWBs enhanced teachers' interactions with their pupils, researchers [1] analysed around 180 Year 5 and 6 numeracy and literacy lessons with and without IWBs using a computerised observation schedule.

Did IWBs make any difference to the teachers' interactions?

The researchers found that using IWBs made little difference to the pattern of interaction that took place in whole-class teaching. They noted how the most frequent teacher interactions were explaining (which took up 28% of the whole class session), followed by asking closed questions, making evaluative comments and directing. The teachers asked significantly more closed questions and fewer open questions in numeracy lessons than in literacy lessons.

They concluded that whilst IWBs are a useful presentational tool in the classroom, the technology does not of itself make teaching more interactive. They suggested for that to happen, it helps if we collaborate with colleagues to refine our practice through observation and coaching, and use data (such as observation and video recordings) from our own classrooms as a starting point for critical reflection.

How can IWBs become tools for enhancing pupils' understanding?

Another study [2] explored how IWB could be used to probe pupils' understanding. It focused on the teaching of quadrilateral definitions through using dynamic geometry software with an IWB. (See figure). Dragging corners A and B altered the size and orientation of the shape, whilst dragging corner C altered the angles. Two girls (Nina and Vanessa) used the software to help them decide whether the shape had been constructed as a square or a rhombus.

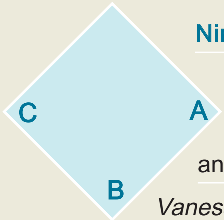
References and resources

[1] Smith, F., Hardman, F. & Higgins, S. (2006). The impact of interactive white-boards on teacher-pupil interaction in the National Literacy and Numeracy Strategies. *British Educational Research Journal 32 (3) pp. 443-457*

[2] Davison, I. (2003) Using an interactive whiteboard to facilitate pupil understanding of quadrilateral definitions. *In Pope, S. (Ed). Proceedings of the British Society for Research into learning Mathematics (23) 1 pp. 13-18.* Available at: *www.bsrlm.org.uk/IPs/ip23-1/BSRLM-IP-23-1-3.pdf*

We suggest resources that can be used with IWBs on the back page.

Square or rhombus?



Nina: It's a rhombus.

Vanessa: Yeah, because look, if you construct it as a rhombus at C, it stays a rhombus (she was dragging point A making the rhombus change in size and orientation, but maintaining the same shape).

Vanessa then used the drag mode to work through the problem and Nina quickly grasped the idea as Vanessa talked.

Vanessa: If it was a rhombus, it would be able to construct into a square because it has the same properties, but you couldn't turn a square into a rhombus because a square … I don't know.

Nina: A constructed square cannot change into a rhombus because it's constructed as a square.

Teacher: So what is it that means it can't be a rhombus?

Vanessa: 'Cos a square has 4 angles that are all 90 degrees. A rhombus doesn't have any really special angles, so it could become a square and still be a rhombus.

The teacher then showed the girls a rectangle, dragged it into a parallelogram, and asked how it was constructed.

Vanessa: Parallelogram because that's not a rectangle. (she drags the parallelogram into a rectangle), but these are still like parallel In a rectangle …

Nina: a rectangle, just like a square has 4 corners of 90 degrees and that's a property of a rectangle … so I think it's constructed as a parallelogram.

The researcher concluded that these pupils' understanding was helped through having the opportunity:

- to see that some quadrilaterals are special examples of other quadrilaterals by dragging the quadrilaterals on their PC and on the IWB
- to create and discuss their own definitions with their teacher and with each other.

Questions, questions, questions

How can you use questions to enhance pupil learning in mathematics?

There is plenty of evidence that questioning is probably *the* most important teaching tool we have. How do you make use of questioning with your pupils? When they were asked, teachers in one study[1] said they used questions for:

- assessing pupils' existing knowledge
- reviewing what pupils learned in a previous lesson
- improving pupil participation in the lesson
- asking children to give their reasoning
- promoting thinking and problem solving
- differentiating by directing specific questions to certain children.

You might like to explore the range of questions you use in your classroom. You could video or tape record a lesson or ask a colleague to listen in and then use the categories above to identify:

- the different kinds of questions you ask
- the categories you use the most and least, and
- any other kinds of questions you ask.

Having identified the pattern of questions you ask, you may like to work on increasing the use you make of certain types of question – both oral and written. For example, you might like to consider how you could design questions to challenge your gifted and talented pupils in particular, or how you could ask your pupils to articulate their reasoning and probe their understanding.

References and resources

[1] Myhill, D., Jones, S. & Hopper, R. (2005) *Talking, listening, learning: Effective talk in the primary classroom.* Open University Press, Maidenhead.

A summary of the research is available on the GTC Research of the Month (RoM) website at: *www.gtce.org.uk/research/romtopics/rom_teachingandlearning/effective_talk/*

A shorter summary of the research is available on The Research Informed Practice Site (TRIPS) at: *www.standards.dfes.gov.uk/research/themes/speakandlisten/talktalk/?version=1*

Guidance materials for supporting pupil learning through talk, produced as part of the research project are available at: *www.people.ex.ac.uk/damyhill/downloads/GuidelineMaterials.doc*

Thinkers: A collection of activities to provoke mathematical thinking. Available from: *www.atm.org.uk/reviews/books/thinkers.html*

How could you increase the challenge for your students?

Here are four key strategies that can help you to make the questions you ask your students more challenging and engaging. We found these evidence-based examples on the National Academy for Gifted and Talented Youth (NAGTY) website in the 'Online CPD in a nutshell' section – 'HOTS not MOTS: higher-order thinking skills, not more of the same', available at: *www.nagty.ac.uk/professional_academy/nutshells/secondary.aspx*

You could set questions which start from the solution and require students to work backwards.	
Instead of:	Find the area & perimeter of a 3x8 rectangle
You could ask:	If the area of a rectangle is 24 cm² and the perimeter is 22cm, what are its dimensions?

You could set questions which can be opened so that they can ask 'what if…..?' questions.	
Instead of:	Find the area & perimeter of a 3x8 rectangle
You could ask:	What if the perimeter is an odd number? What if the perimeter is larger than the area? What if the perimeter and the area are identical?

You could set questions which have a variety of solutions, then ask students to justify that they have found them all.	
Instead of:	Find the area & perimeter of a 3x8 rectangle
You could ask:	Find a rectangle which has unit sides and a perimeter of 100. How many answers are there and how do you know you've got them all?

You could set questions which can be solved in more than one way so that they can ask students to elaborate on their strategies.	
Instead of:	Find the area and perimeter of a 3x8 rectangle
You could ask:	If the area of a rectangle is 24 cm² and the perimeter is 22cm, what are its dimensions? How did you do it?

How do you explore your pupils' reasoning and probe their understanding?

To help you consider how you ask pupils to share their reasoning, you may find it useful to record a lesson – or part of it – or ask a colleague to observe your lesson. You could focus on the types of questions you ask, distinguishing between factual questions requiring recall of knowledge and process questions that require pupils to explain their reasoning and the answers your pupils give. You could record your observations in a table:

Question	Type of question (factual or process)	Pupil's response
What answer did you get?	Factual	
How did you work that out?	Process	
Can you explain why?	Process	

What proportion of your questions required recall of facts and what proportion required pupils to share their reasoning? What do your pupils' answers tell you about their individual levels of understanding?

Depending on your results, you might like to work with a colleague on turning your factual questions into process questions so that you probe your pupils' understanding more often and more deeply. You might also like to list possible answers your pupils may give for some of the questions, including possible misconceptions, and discuss with your colleague how you might respond.

Fractions: difficult but crucial

How might we help pupils to learn difficult concepts?

Fractions is one of those concepts that many pupils find difficult. A reason for their difficulty is the relative nature of fractions: that the same fraction can refer to different quantities (for example 1/2 of 8 and 1/2 of 12 are different) and different fractions can be equivalent because they refer to the same quantity (1/3 and 3/9 for example). [1] Researchers who explored ways in which teachers can help their pupils grasp the concept noticed how they were better at solving some kinds of fractions problems than others. They also found that when teachers used the approaches the pupils were naturally better at as a starting point for teaching the concept, the pupils made more progress.

What kind of fractions problems were the pupils better at solving?

The researchers gave pupils questions about equivalent fractions presented in two different ways. In one test, pupils were asked to solve equivalent fractions problems in part-whole situations – the way that fractions are often introduced in primary schools. For example, one question asked pupils to colour 2/3 of two shapes, one divided into 6 and the other divided into 9 equal parts. This involved thinking about equivalent fractions because the pupils had to colour in 4/6 and 6/9 respectively. In the other test, pupils were given fractions questions which focused on sharing, such as, is one cake shared between three people the same as two cakes shared between six people?

Surprisingly, the pupils were much better at solving the sharing problems (for which they had received no teaching) than the part-whole problems (the style of problem they were often given). The researchers explored this further by working with small groups of pupils as they solved problems such as finding different ways of sharing two pizzas equally between six children. They found that when they asked the pupils whether the children would eat the same amount either way, the children's arguments were often based on the logic of division. Essentially, the pupils argued that if the pizzas were shared fairly and completely, the way in which the pizzas were cut up did not matter. The pupils also understood that the more people there were sharing something, the less each one got (i.e. that 1/5 of a pizza is smaller than 1/3).

How did the researchers use their insights to inform teaching?

The researchers asked five teachers to use sharing problems with their classes for 5-8 sessions. They then compared their pupils' performance before and after the experiment with other pupils in who had received their usual instruction on fractions. Before the experiment there was no difference between the groups, but at the end of the teaching period the groups that had been taught using sharing problems outperformed the comparison groups in the same fractions test, and they were still performing better eight weeks later.

But the teaching experiment also revealed that pupils' initial learning of fractions was to some extent context specific. For example, pupils who understood that 1/3 and 2/6 were equivalent fractions when discussing the division of pizzas, did not understand that a mixture of one glass of orange concentrate with two glasses of water had the same taste as another where two glasses of orange concentrate and four glasses of water were used. This showed the importance of extending pupils' knowledge across situations.

How could you put this research to work?

This study showed that when solving equivalent fractions, problems pupils found sharing situations easier than part-whole and that building on pupils' intuitive understanding provided a sound starting point for teaching. The study also showed the importance of planning lessons that enable pupils to explore a concept in a range of situations.

- Have you noticed whether your pupils find it easier to solve fraction problems based on sharing than on part-whole relationships? Can you find examples in text books etc that illustrate solving fraction problems by sharing, which you could use with your pupils?
- Could you work with your colleagues and/or your pupils to discover different methods of teaching difficult concepts and monitor how your pupils respond to these alternative strategies?
- Would you find it helpful to work with colleagues to scan books and web resources for examples of problems set in a range of contexts? Could you arrange for your pupils to work with you in small groups to discuss the strategies they use as they solve the problems?

References and resources

[1] Nunes, T., Bryant, P. & Hurry, J. (2006) Fractions: Difficult but crucial in mathematics learning. Teaching and Learning Research Programme Research Briefing no.13. Available at: www.tlrp.org/pub/documents/no13_nunes.pdf

The Maths4Life website has a section devoted to teaching fractions at: www.maths4life.org/content.asp?CategoryID=1072
It includes a downloadable booklet and suggestions for activities designed to help learners to relate fractions to other mathematical concepts.

And now for something completely different...

Helping children think: gaze aversion and teaching

Gaze aversion, which is the act of looking away from something, has been found to benefit adults when dealing with difficult problem-solving activities. So researchers at Stirling University decided to investigate the impact of using gaze aversion with children to see how it would affect their problem-solving abilities.

The researchers tested the potential benefits – as a teaching strategy – of teaching five year olds to use gaze aversion when working on arithmetic and verbal reasoning questions and found that they performed significantly better on more challenging questions than the control group. The team also found that pupils' use of gaze aversion naturally increases during their first year of schooling.

Twenty Year 1 pupils were selected from primary schools in Stirlingshire, of which ten were encouraged to look away while they were thinking whilst the other ten were not given this instruction and acted as a control group. The pupils were videoed while they were asked verbal and arithmetic questions. Half of the questions were easy and half were moderately difficult.

The test group who were using gaze aversion were found to be more accurate than the control group in answering both verbal-reasoning and mental arithmetic questions. The difference was especially noticeable for harder questions, but when questions were too easy or trivial, gaze aversion did not significantly help pupils.

Results for the use of gaze aversion and answer accuracy

	Use of gaze aversion	Accuracy in answers to questions
Test group	52.5%	72.5%
Control group	34.7%	55.9%

Gaze aversion seems to work by giving pupils the opportunity to concentrate on the problem without immediate distractions. According to the researchers, "Given that five-year old children could be readily trained to increase their use of gaze aversion, coupled with the finding that this training could significantly benefit performance, encouragement of gaze aversion while the child is thinking appears to be a simple, yet effective way in which to significantly improve a five-year-old child's cognitive performance." The extent of a child's gaze aversion could also serve as a useful tool for identifying when children are engaged in cognitive activity.

The researchers also investigated the spontaneous or natural use of gaze aversion in Year 1 pupils during their first year at school. The pupils were not introduced to gaze aversion or given any training in its use because the researchers were interested to see how gaze aversion changed naturally. The investigation found that pupils had low level of gaze aversion use at the start of Year 1, but that this increased during the year and was significantly higher at the end of the year. This could either be explained in terms of age-related developmental change or because of increased exposure to teaching and learning in school.

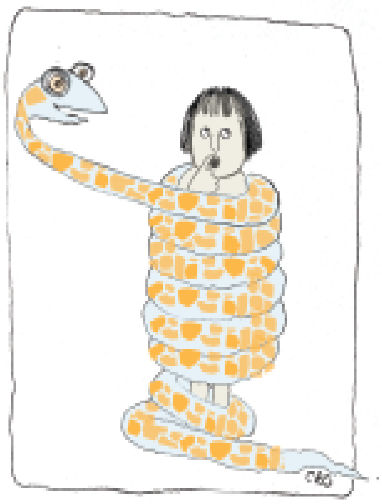
Here are some examples of the types of questions used in the test

Verbal Questions	
Easy	What is a dog?, Tell me the colour of the sea
Moderately difficult	What is a telescope?, Tell me the 7 days of the week

Arithmetic Questions	
Easy	1 + 1?, 2 – 1?, Count to 10
Moderately difficult	4 + 4?, 2 x 3?, Count backwards from 10

What might this mean for you?

The study explains that gaze aversion has been found to be a beneficial strategy for dealing with cognitive challenges for young children right through to adults. If this is true of adults and young children, you might want to focus on pupils (of any age) tackling challenging mental problems to see if they use gaze aversion. Can you identify students who are using it often or less often? Perhaps it might be worth experimenting by encouraging students to use gaze aversion in problem solving?



THIS IS ONE PROBLEM I NEED TO SOLVE FIRST!

Reference

Phelps, F. G., Doherty-Sneddon, G., and Warnock, H. (2006) British Journal of Developmental Psychology Vol.24, pp.577-588

Further information about gaze aversion and an explanation of the research undertaken by Stirling University can be found at:
www.psychology.stir.ac.uk/staff/lcalderwood/GazeAversionResearch.htm

Mathematics resources on the web

www.ncetm.org.uk/find

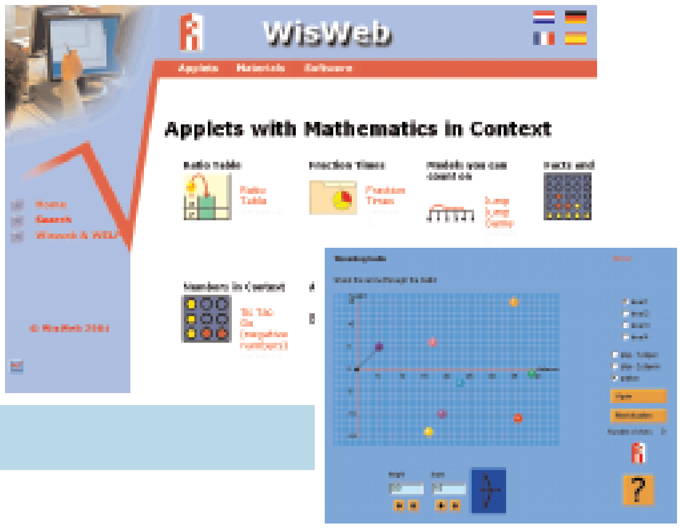
The resources on the National Centre for Excellence in the Teaching of Mathematics (NCETM) Portal are arranged into five collections: web links to online resources for you to use with your learners, articles for you to discuss with your colleagues, schemes of work and lesson plans to support your teaching and professional development, case studies of good practice, and research papers that investigate wider issues in mathematics education. You will also find discussions and articles from, and about teachers trying out some ideas from research in the Communities and Blogs. You might like to add your own experiences.

www.nrich.maths.org

You'll find Cambridge University's NRICH website is packed with exciting mathematics problems and games for you to use with your pupils. The activities will help you make the most of technology whilst providing a focus for group work by giving your pupils opportunities to explore and engage with a range of mathematical ideas. Each month there's a new selection of puzzles on a theme, with something for every age group. There's also a range of articles that will give you even more ideas for making maths relevant, interesting and practical. Many of these also explain the research evidence and/or the rationale underpinning them, such as why pupils have difficulties with division and how you can help pupils understand difficult concepts like fractions.

www.fi.uu.nl/wisweb/en/

This Dutch website features a range of 'applets' – small computer programmes that run over the internet, which can be used by teachers using an interactive whiteboard and/or pupils working individually or in groups at the computer. They offer primary and secondary aged pupils, as well as vocational learners opportunities to investigate mathematical ideas in an interactive and dynamic manner. Our picture shows some of the 'applets with mathematics in context'. 'Applets with algebra courseware on linearity' are also available.



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